Persuasion with Endogenous Misspecified Beliefs*

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Abstract

We study a two-action, two-state pure persuasion game in which the receiver has non-rational expectations. The sender can add ambiguity to his message by pooling it with other messages. This can be likened to selective redaction of the original message. The receiver knows the sender's message strategy but not his redaction strategy, and uses only the former to draw inferences from the redacted message. We characterize the highest probability of persuasion attainable by the sender under these conditions.

Keywords: Persuasion, Misspecified beliefs, Non-rational expectations.

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1 Introduction

The standard assumption in models of strategic information transmission is that the uninformed party (the receiver) perfectly observes the informed party's (the sender's) message, and has perfect knowledge of the distribution of messages conditional on the state. In other words, the receiver has *perfect* knowledge of *both* the *equilibrium strategy* of the sender and the *realized message*. This short paper examines the implications of relaxing these assumptions.

Our objective is motivated by the observation that in many real-life circumstances, the sender's message is not directly transmitted to the receiver. Rather, the receiver obtains the message through an intermediary. For example, oftentimes politicians' statements in closed doors are leaked to the media, where the source of the leak has discretion over the precision level of his information (e.g., a precise statement about imposing tariffs on another country may be leaked as a vague statement about taking economic measures against it). Similarly, information on statements made by an executive in top management meetings tend to reach workers via other members in the firm, who may add noise to the original message (e.g., a statement about laying off workers may be transmitted as a general message about taking measures to increase efficiency). Finally, the sender himself may decide to conceal parts of his statements (e.g., by redacting selected passages in a written document).

These situations appear inherently different than those in which the receiver directly observes the sender's message. This is particularly true when the intermediary who redacts or adds noise to the original message has a conflict of interests with the receiver. In that case, the intermediary may decide strategically how coarsely he will describe the messages. However, this can only have an effect on the receiver's beliefs and behavior if he has *non*-rational expectations. Under rational expectations, the receiver knows exactly the joint distribution over states, messages *and* degrees of coarseness; therefore, he would be able to make correct inferences from the description of the message that he receives (thereby "undoing" their coarseness), as long as the set of possible noisy/redacted messages is at least as rich as the original message space.

It follows that in order to investigate the implications of incomplete receiver knowledge of the sender's message, we must also relax the rational-expectations assumption. We explore this direction in the context of a pure persuasion game. An informed sender sends a message in order to persuade an uninformed receiver to choose a particular action. The receiver wants to choose that action only in one of the states, while the sender wants him to choose that action in all states. In contrast to the standard persuasion framework, in our model the receiver does not directly observe the message sent by the sender. Whenever the sender sends a message, he (or an intermediary who acts as his proxy) discloses to the receiver a *set of messages* that contains the realized message. In other words, the receiver obtains an *information set* that includes the actual message, but possibly additional ones. It is as if the sender tells the receiver: "One of the following messages was sent". A more realistic image is selective redaction of the message's content. Thus, the sender cannot lie, but he can strategically add ambiguity to his message.

We assume that the receiver has correct expectations about the distribution of messages in each state. Given the set of messages reported by the sender, the receiver updates his beliefs about the state according to this distribution. This aspect of his beliefs is consistent with rational expectations. Where we depart from rational expectations is by assuming that the receiver does *not* know how the sender conditions the reported set of messages on the state. In other words, he does not know the sender's redaction strategy. Therefore, the receiver does not draw any inference from the fact that the sender chose to report one set of messages and not another. Thus, while the standard approach of rational expectations may be viewed as one extreme, where each player has perfect knowledge and understanding of the equilibrium, our approach of boundedly-rational expectations is on the other extreme, where a player's knowledge of the equilibrium is based solely on his prior beliefs and any information he is given.

To sum up, the receiver has rational expectations about the state-message relationship, but lacks rational expectations about the redaction strategy. We characterize the maximal probability of persuasion that the sender can achieve under these conditions, as a function of the prior beliefs and the size of the message space n. In particular, when the prior on the state in which the receiver and sender have common interests is at least $1/n$, the sender can persuade the receiver to choose his favorite action in *both* states. Thus, when this condition is satisfied, the sender can achieve his first-best even when he cannot commit to a strategy.

This short paper follows up on our earlier work, Eliaz, Spiegler and Thysen (2020), which also analyzed how a sender can manipulate a receiver's misspecified beliefs in the absence of rational expectations. In that paper, we used a different terminology and motivation for the receiver's non-rational expectations: The intermediary does not conceal or add noise to the sender's message, but rather provides *partial statistical data* that enables the receiver to draw partial inferences from the sender's (fully observed) message. Using the current paper's terminology, the basic model of Eliaz, Spiegler and Thysen (2020) corresponds to restricting the domain of feasible "redaction strategies" available to the sender (or his proxy): The message is multi-dimensional, and the sender redacts some of its components. However, the "data provision" interpretation enables Eliaz, Spiegler and Thysen (2020) to examine alternative sender-receiver models that lie outside the scope of the present paper - e.g., a model in which the sender can provide multiple datasets about different message components.

The current paper also joins a recent small literature on persuasion with non-rational expectations, which include de Barreda, Levy and Razin (2018), Galperti (2019), Glazer and Rubinstein (2012, 2014), Mullainathan, Schwartzstein, and Shleifer (2008) and Schwartzstein and Sunderam (2021). A more detailed discussion of these papers appears in Eliaz, Spiegler and Thysen (2020).

The receiver's information structure in our model is closely related to Jehiel's (2005) notion of analogy-based expectations equilibrium (ABEE). According to this concept, players form coarse beliefs that are measurable with respect to an "analogy partition" of the possible states of the world. Our model can be viewed as an extensive game in which the sender chooses the message as well as the receiver's analogy partition, and the solution concept is ABEE. However, since the only partition cell that matters for the receiver's belief is the one that contains the actual message, the ABEE formulation is less economical in this context than the our own. Other works that applied ABEE to sender-receiver games include Jehiel and Koessler (2008) and Hagenbach and Koessler (2019). Jehiel (2011) studies endogenous creation of analogy partitions by an auction designer. Finally, the receiver's behavior in our model is also somewhat similar to the "problem solver" in Glazer and Rubinstein (2019).

The idea that an informed sender may wish to manipulate the beliefs of an uninformed receiver by disclosing some event that contains the true state, dates back to Grossman (1981) and Milgrom (1981). These papers show that when the receiver has rational expectations and the sender's preferences are monotone in the receiver's action, information unravels, i.e., the sender discloses the state in equilibrium. Milgrom and Roberts (1986) consider the case where some proportion of receivers are naive in that they take the sender's message at face value. Although the sender in this case will not fully reveal the state, receivers with rational expectations will still be able to infer from the true state from the sender's strategy. Okuno-Fujiwara, Postlewaite and Suxumura (1989) provide general sufficient conditions for information unraveling in games of incomplete information.

2 A Model

There are two states of nature, Y and N. The common prior on state Y is $\pi < \frac{1}{2}$. The receiver has two possible actions, y and n. The receiver's payoff depends on the action he takes and the state. His payoff is 1 if he chooses y in Y or n in N, and it is zero otherwise. Hence, based on his prior alone, the receiver's optimal action is n. The sender's payoff depends only on the receiver's action: It is equal to 1 if Y is chosen and it is zero otherwise.

Let M be a finite set consisting of n feasible messages. For every $m \in M$, let $\mathcal{I}(m)$ be *all* subsets $I \subseteq M$ that include m. The sender's feasible action set (independently of the state) is $\mathcal{A} = \{(m, I) \mid m \in M, I \in \mathcal{I}(m)\}$. The meaning of an action (m, I) is that m is the sender's actual message and I is the receiver's information set - i.e. he only learns that $m \in I$.

The sender commits to a strategy $\sigma : \Theta \to \Delta(\mathcal{A})$. Let $\sigma(m, I \mid \theta)$ denote the probability that the strategy assigns to the action (m, I) in state θ. With slight abuse of notation, define $\sigma(m \mid \theta) = \sum_I \sigma(m, I \mid \theta)$, which is the probability that the message m is played in θ. We refer to $\sigma(m \mid \theta)$ as the sender's message sending strategy. The receiver uses naive Bayesian updating to form his posterior belief. That is, given the sender's strategy σ, when the action (m, I) is realized, the receiver's posterior belief about the likelihood that $\theta = Y$ is given by:

$$P_\sigma(m, I) = \frac{\pi \sum_{m' \in I} \sigma(m' \mid \theta = Y)}{\pi \sum_{m' \in I} \sigma(m' \mid \theta = Y) + (1 - \pi) \sum_{m' \in I} \sigma(m' \mid \theta = N)}$$

The receiver's subjective likelihood ratio of (m, I) is therefore

$$\rho_\sigma(m, I) = \frac{\sum_{m' \in I} \sigma(m' \mid \theta = Y)}{\sum_{m' \in I} \sigma(m' \mid \theta = N)}$$

Given a strategy σ, the receiver chooses y in response to (m, I) if and only if $P_\sigma(m, I) \geq \frac{1}{2}$. Equivalently, the receiver chooses y if and only if $\rho_\sigma(m, I) \geq$

$(1-\pi)/\pi$.

To illustrate the model, let $n = 3$, $\pi = \frac{1}{3}$ and σ^* be the following strategy. In state Y the sender plays $(m_1, \{m_1\})$ with certainty. In state N, he uniformly randomizes over the two actions $(m_2, \{m_1, m_2\})$ and $(m_3, \{m_1, m_3\})$. When the sender chooses $(m_1, \{m_1\})$, the receiver is informed that message m_1 was sent. Since m_1 is only sent in state Y, the receiver concludes that the state is Y, i.e., $P_{\sigma^*}(m_1, \{m_1\}) = 1$, and will therefore choose y. In contrast, when action $(m_2, \{m_1, m_2\})$ is taken, the receiver is informed that the event $\{m_1, m_2\}$ occurred. This event occurs with probability one in state Y and with probability 0.5 in state N. Hence, the receiver's induced posterior belief $P_{\sigma^*}(m_2, \{m_1, m_2\})$ is equal to

$$\frac{(\frac{1}{3})(1)}{(\frac{1}{3})(1) + (\frac{2}{3})(\frac{1}{2})} = \frac{1}{2}$$

and he will therefore choose y. In contrast, if the receiver had rational expectations he would know that the sender reports the event $\{m_1, m_2\}$ only in state N, and would therefore conclude that the state is N with certainty.

As this example illustrates, the sender essentially chooses an information structure for the receiver. However, the sender can manipulate the receiver's beliefs by exploiting the fact that the receiver does not have rational expectations, and that given the realized information set, he bases his inference only on the message sending strategy. This is done by giving the receiver a *non-partitional* information structure. In the above example, when message m_2 is sent in state N the receiver has the information set $\{m_1, m_2\}$. But when message m_1 is sent, the receiver has the information set $\{m_1\}$, and when m_3 is sent the receiver has the information set $\{m_1, m_3\}$.

Non-partitional information structures violate the introspection axioms that characterize the standard epistemic model of possibility correspondences that underlies Harsanyi's model of games with incomplete information (see Rubinstein (1998, Ch. 3) and Geanakoplos (1989)). In the present context, they mean that the receiver draws correct statistical inferences from learning that a particular event has occurred, but makes no inference from the fact that other events have not occurred. In particular, the receiver does not draw any inference from the realized information set itself. Hence, the information set that accompanies one message does not affect the receiver's inference about the state when he receives another message that is accompanied by a different information set. For instance, in the previous example, the information set $\{m_1, m_2\}$ that accompanied the message m_2 does not affect the receiver's beliefs when he receives the message m_1 together with the information set $\{m_1\}$. Thus, if the sender modifies his strategy by changing the information set that accompanies a particular message, this does not affect the receiver's inferences from other messages.

The sender's objective is to choose σ that maximizes the probability that the receiver chooses y, subject to the constraint that the receiver's action is optimal given his posterior belief.

3 The Result

Our main result characterizes the maximal probability of persuasion as a function of the relation between the prior π and the total number of messages n.

Proposition 1
(*i*) *Let* $\pi \geq 1/n$. *Then, the following strategy attains full persuasion. In state* Y*, the sender plays* $(m_1, \{m_1\})$ *with probability one. In state* N*, he uniformly randomizes over the* $n-1$ *actions* $(m_2, \{m_1, m_2\}), ..., (m_n, \{m_1, m_n\})$.

(*ii*) *Let* $\pi < 1/n$. *Then, the maximal probability of persuasion is* $\pi(n-1)$*, implemented by the following strategy. In state* Y*, the sender plays* $(m_1, \{m_1\})$ *with probability one. In state* N*, he assigns probability* $\pi/(1-\pi)$ *to each of the the* $n-2$ *actions* $(m_2, \{m_1, m_2\}), ..., (m_{n-1}, \{m_1, m_{n-1}\})$*, and probability*

$1-(n-2)\pi/(1-\pi)$ *to the action* $(m_n, \{m_1, m_n\})$.[1]

Thus, in the optimal strategy, the sender sends a single message in the Y state and adds no noise or ambiguity. In contrast, he randomizes over messages in the N state and adds ambiguity to these messages, by accompanying them with information sets that pool them with the Y-state message. Note that this ambiguity is "minimal", in the sense that no information ever contains more than two messages. Indeed, the receiver would benefit from an intervention that forces the sender to add ambiguity by increasing the size of information sets. To see why, note that the sender cannot benefit from pooling messages that are all sent in the N state. Therefore, larger information sets imply that multiple messages are sent in the Y state. However, this effectively reduces the number of messages that are available to the sender and therefore harms his ability to persuade.

Compare Proposition 1 with the basic model in Eliaz, Spiegler and Thysen (2020). Using the current terminology, that model corresponds to an environment in which $M = \{0,1\}^K$ and $\mathcal{I}(m)$ is the collection of all sets $I = \{m' \mid m'_D = m_D\}$ for some non-empty $D \subseteq \{1, ..., K\}$. (The models in Sections 4.2 and 5 of that paper cannot be translated into the current language.) In that case, full persuasion is only attained for

$$\pi \geq \frac{1}{1 + \binom{K}{\lfloor K/2 \rfloor}} > \frac{1}{2^K}$$

Thus, when we impose natural structure on the set of feasible information sets, this has a substantial effect on the sender's ability to persuade the receiver.

[1]The information set accompanying m_n can be replaced with any $I \in \mathcal{I}(m_n)$, as none of the resulting pairs (m_n, I) persuades the receiver anyway. We selected the specific information set $\{m_1, m_n\}$ because it streamlines the proof.

Proof of Proposition 1

We first argue that the strategy σ outlined in the Proposition persuades the receiver with probability one when $\pi \geq 1/n$ and with probability $(n-1)\pi$ when $\pi < 1/n$. To see this, note that when $\pi \geq 1/n$, then $P_\sigma(m_1, \{m_1\}) = 1$;

$$P_\sigma(m_i, \{m_1, m_i\}) = \frac{\pi(n-1)}{\pi(n-2)+1} \geq 1/2$$

for $i = 2, \ldots, n$. When $\pi < 1/n$, then $P_\sigma(m_1, \{m_1\}) = 1$; $P_\sigma(m_i, \{m_1, m_i\}) = 1/2$ for $i = 2, \ldots, n-1$ and

$$P_\sigma(m_n, \{m_1, m_n\}) = \frac{\pi}{1-(n-2)\pi} < 1/2.$$

We now proceed to show that no other strategy achieves a higher probability of persuasion. Let σ be an optimal sender strategy.

Lemma 1 *Without loss of generality, we can restrict attention to strategies that accompany each message m with a unique information set $I(m)$.*

Proof. Suppose that the pairs $(m, I), (m, I')$ are both played with positive probability under σ, such that $I \neq I'$ and $\rho_\sigma(m, I) \geq \rho_\sigma(m, I')$. Let $\hat{\sigma}$ be a strategy that differs from σ only by replacing every occurrence of (m, I') with (m, I). Since the deviation does not change the distribution of messages conditional on each state, it leaves $\rho_\sigma(m, I)$ and $\rho_\sigma(m, I')$ unchanged, and it does not affect the likelihood ratio of any other report. Therefore, the deviation weakly raises the probability of persuasion. ■

Henceforth, we restrict attention to strategies in which each $m \in M$ that is sent with positive probability is paired with a unique information structure $I(m)$. Define J as the set of messages m for which

$$\frac{\sigma(m \mid \theta = Y)}{\sigma(m \mid \theta = N)} > \frac{1-\pi}{\pi}$$

By the definition, the receiver would be persuaded by the pair $(m, \{m\})$ for every $m \in J$. Since the sender can always select $I(m) = \{m\}$, it follows that $\rho_\sigma(m, I(m)) \geq (1-\pi)/\pi$ for every $m \in J$.

Lemma 2 *Without loss of generality, we can set $I(m) = J \cup \{m\}$ for every message m that is sent with positive probability.*

Proof. For any action $(m, I(m))$ that is played with positive probability and does *not* persuade the receiver, it must be the case that $\rho_\sigma(m, I) < (1-\pi)/\pi$ for *all* $I \in \mathcal{I}(m)$. Therefore, we can select $I(m) = J \cup \{m\}$ without loss of generality in this case. It remains to show that $\rho_\sigma(m, J \cup \{m\}) \geq (1-\pi)/\pi$ for every message m for which $\rho_\sigma(m, I(m)) \geq (1-\pi)/\pi$.

Let $(m, I(m))$ be an action that is played with positive probability and persuades the receiver. Then,

$$\pi \sum_{m' \in I(m)} \sigma(m', I(m') \mid \theta = Y) \geq (1-\pi) \sum_{m' \in I(m)} \sigma(m', I(m') \mid \theta = N) \quad (1)$$

Suppose, in contradiction to the claim, that $I(m) \neq J \cup \{m\}$. In particular, suppose there is a message $\tilde{m} \in J - I(m)$. By the definition of J,

$$\pi\sigma(\tilde{m}, I(\tilde{m}) \mid \theta = Y) > (1-\pi)\sigma(\tilde{m}, I(\tilde{m}) \mid \theta = N)$$

Adding these two inequalities, we get

$$\frac{\sum_{m' \in I(m) \cup \{\tilde{m}\}} \sigma(m', I(m') \mid \theta = Y)}{\sum_{m' \in I(m) \cup \{\tilde{m}\}} \sigma(m', I(m') \mid \theta = N)} > \frac{1-\pi}{\pi}$$

Therefore, we can add $\tilde{m}$ to $I(m)$ and the action $(m, I(m) \cup \{\tilde{m}\})$ will still persuade the receiver.

Now suppose there is a message $\hat{m} \in I(m) - J$. By the definition of J,

$$\pi\sigma(\hat{m}, I(\hat{m}) \mid \theta = Y) \leq (1-\pi)\sigma(\hat{m}, I(\hat{m}) \mid \theta = N)$$

Subtracting this inequality from (1) and rearranging, we get

$$\frac{\sum_{m' \in I(m)-\{\hat{m}\}} \sigma(m', I(m') \mid \theta = Y)}{\sum_{m' \in I(m)-\{\hat{m}\}} \sigma(m', I(m') \mid \theta = N)} \geq \frac{1-\pi}{\pi}$$

Therefore, the receiver would also be persuaded by the action $(m, I(m) - \{\hat{m}\})$.

We can repeat this process of adding or eliminating elements, until $(m, I(m))$ is replaced with $(m, J \cup \{m\})$ and the probability of persuasion is unchanged. ■

Lemma 3 *If σ is an optimal strategy, then J is non-empty.*

Proof. If $J = \emptyset$, then by Lemma 2 it is without loss to assume that each message m is sent with $\{m\}$. Since such a strategy endows the receiver with rational expectations, it implies that the maximal probability of persuasion is 2π. But then σ cannot be optimal, since we have already identified a strategy that achieves a higher probability of persuasion. ■

We are now ready to derive the upper bound on persuasion. Define H as the set of messages $m \notin J$ that are played with positive probability such that $(m, I\{m\})$ persuades the receiver. Assume that $I\{m\} = J \cup \{m\}$ persuades the receiver. By Lemma 2 this is without loss. Then, for every $m \in H$, we have

$$\begin{aligned} \rho_\sigma(m, J \cup \{m\}) &= \frac{\sum_{m' \in J \cup \{m\}} \sigma(m', J \cup \{m'\} \mid \theta = Y)}{\sum_{m' \in J \cup \{m\}} \sigma(m', J \cup \{m\} \mid \theta = N)} \\ &= \frac{\sum_{m' \in J \cup \{m\}} \sigma(m', J \cup \{m'\} \mid \theta = Y)}{\sum_{m' \in J} \sigma(m', J \mid \theta = N) + \sigma(m, J \cup \{m\} \mid \theta = N)} \geq \frac{1-\pi}{\pi} \end{aligned} \tag{2}$$

Rearranging the inequality, we obtain an upper bound on the probability that the action $(m, J \cup \{m\})$ is played in state N:

$$\sigma(m, J \cup \{m\} \mid \theta = N) \leq \frac{\pi}{1-\pi} \sum_{m' \in J \cup \{m\}} \sigma(m', J \cup \{m'\} \mid \theta = Y) - \sum_{m' \in J} \sigma(m', J \mid \theta = N)$$
$$\leq \frac{\pi}{1-\pi} - \sum_{m' \in J} \sigma(m', J \mid \theta = N). \tag{3}$$

Note, that (3) is the same for every $m \in H$. Therefore, the probability of persuasion in state N is bounded above as follows

$$\sum_{m \in H \cup J} \sigma(m, J \cup \{m\} \mid \theta = N)$$
$$\leq |H| \left[\frac{\pi}{1-\pi} - \sum_{m' \in J} \sigma(m', J \mid \theta = N) \right] + \sum_{m' \in J} \sigma(m', J \mid \theta = N)$$
$$\leq |H| \frac{\pi}{1-\pi} - (|H| - 1) \sum_{m' \in J} \sigma(m', J \mid \theta = N). \tag{4}$$

By definition of J we have $\sigma(m', J \mid \theta = N) < \pi/(1-\pi)$. This implies that the upper bound on persuasion is increasing in $|H|$ and thus decreasing in $\sigma(m', J \mid \theta = N)$ for every $m' \in J$ when $n > 2$. By Lemma 3, $|H| \leq n - 1$ since $J \neq \emptyset$. Furthermore, if there exists an action that does not persuade the receiver, then $|H| \leq n - 2$. This will always be the case when $(n-1)\pi/(1-\pi) < 1$.

It follows that the upper bound on the overall probability is

$$\begin{array}{ll} \pi + (1-\pi)(n-1)\frac{\pi}{1-\pi} & \textit{when} \quad \pi \geq \frac{1}{n} \\ \pi + (1-\pi)(n-2)\frac{\pi}{1-\pi} & \textit{when} \quad \pi < \frac{1}{n} \end{array}$$

This completes the proof. ■

References

[1] Ines De Barreda, Gilat Levy, and Ronny Razin. Persuasion with correlation neglect. mimeo, 2018.

[2] Kfir Eliaz, Rani Spiegler, and Heidi C. Thysen. Strategic Interpretations. *Journal of Economic Theory*, forthcoming.

[3] Simon Galperti. Persuasion: The art of changing worldviews. *American Economic Review*, 109(3):996-1031, 2019.

[4] John Geanakoplos. Game Theory Without Partitions, and Applications to Speculation and Consensus. SFI working paper 1990–018.

[5] Jacob Glazer and Ariel Rubinstein. A model of persuasion with boundedly rational agents. *Journal of Political Economy*, 120(6):1057–1082, 2012.

[6] Jacob Glazer and Ariel Rubenstein. Complex Questionnaires. *Econometrica*, 82:1529-1541, 2014.

[7] Jacob Glazer and Ariel Rubinstein. Coordinating with a "Problem Solver". *Management Science*, 65:2813–2819, 2019.

[8] Sanford J. Grossman. The informational role of warranties and private disclosure about product quality. *The Journal of Law and Economics*, 24:461-483, 1981.

[9] Jeanne Hagenbach and Frédéric Koessler. Cheap talk with coarse understanding. *Games and Economic Behavior*, 124:105-121, 2020.

[10] Philippe Jehiel. Analogy-based expectation equilibrium. *Journal of Economic theory*, 123(2):81–104, 2005.

[11] Philippe Jehiel. Manipulative auction design. *Theoretical economics*, 6(2):185–217, 2011.

[12] Philippe Jehiel and Frédéric Koessler. Revisiting games of incomplete information with analogy-based expectations. *Games and Economic Behavior*, 62(2):533–557, 2008.

[13] Paul R. Milgrom. Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 380-391, 1981.

[14] Paul R. Milgrom, and John Roberts. Price and Advertising Signals of Product Quality. *Journal of Political Economy*, 94(4):795-821, 1986.

[15] Sendhil Mullainathan, Joshua Schwartzstein, and Andrei Shleifer. Coarse thinking and persuasion. *The Quarterly journal of economics*, 123(2):577–619, 2008.

[16] Masahiro Okuno-Fujiwara, Andrew Postlewaite, Kotaro Suxumura. Strategic Information Revelation. *The Review of Economic Studies*, 57:25-47, 1990.

[17] Ariel Rubinstein. Modeling Bounded Rationality. MIT Press. 1998.

[18] Joshua Schwartzstein and Adi Sunderam. Using Models to Persuade. *American Economic Review*, 111(1):276-323, 2021.